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STRESS-STRAIN ANALYSIS OF AN OPEN-ENDED THICK CYLINDER OF STRAIN-HARDENED MATERIAL BY AUXILIARY ANGLE METHOD

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ABSTRACT

This work aims at the determination of the stresses and strains from the analytical solution of an open-ended thick cylinder of strain-hardened material by auxiliary angle method by using small deformation theory and von Mises criterion. This solution submits a new analytical method for the elastic-plastic stresses, strains and displacements. Study of the strength design by plastic-limit pressure and corresponding displacement for various radius ratios of the cylinder are presented.

Keywords: Strain-hardened Material, Auxiliary Angle, Von Mises Criterion

1. INTRODUCTION

Design of open-ended thick cylinder is based on the autofrettage analysis for full use of the capacity of the material. Prediction of the residual hoop stress distribution in autofrettaged thick-walled tubing of high strength low-alloy steel with a diameter ratio is presented in [1]. Plastic stress and strain distribution induced by a single over-pressure cycle in a cylindrical vessel considering a linear hardening law and Ramberg-Osgood law for an ideal elastic-perfectly plastic-material are presented and residual stress fields are numerically evaluated, during unloading phase, for stress and strain in [2]. Optimum radius of elastic-plastic juncture, influence of autofrettage on stress distribution and load-bearing capacity of a cylinder is studied and optimum pressure in autofrettage technology is presented in [3]. A numerical procedure for modeling autofrettage of thick-walled cylinders with Bauchinger effect as a function of prior plastic strain and von Mises criterion is presented in [4]. Auxiliary variable method for infinite plate with circular hole is furnished in [5] as well as for thick-walled cylinder made of strain-hardening material for stress-strain is furnished in [6].

Scope is to find out the elastic-plastic stresses and strains from the general analytical solution of a strain-hardened open-ended thick cylinder by auxiliary angle method using small deformation theory and von Mises criterion. It is a new analytical method for the elastic-plastic stress and strain analysis and the strength design by plastic-limit. Setting the strain-hardening index of the material equal to zero, the solution and the results are reduced to those of open-ended thick cylinder of elastic-perfectly plastic-material [6, 7]. A numerical solution is also presented to study the effect of the elastic-limit pressure and the plastic-limit (full-yield) pressure for an open-ended thick cylinder of elastic-perfectly plastic-material and the strain-hardened materials, for the different radius ratios of the cylinder on the limiting load-carrying capacity of an open-ended thick cylinder.

2. ANALYSIS OF THE CYLINDER

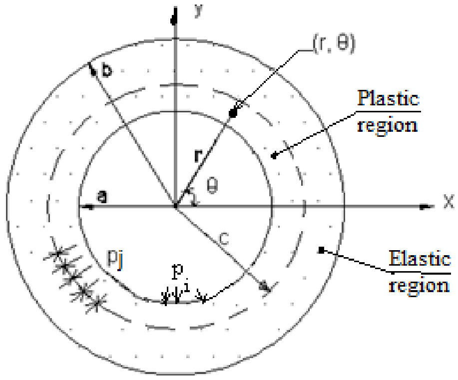

Fig 1. An open-ended thick cylinder

An open-ended thick cylinder of strain-hardened, homogeneous and incompressible material, as shown in figure1, of inside radius, a, and outside radius, b, subjected to small strain and internal pressure, p_i symmetrically with a cylindrical interface of radius, c ($a \le c \le b$) concentric with the inside and outside surfaces of the cylinder, inside of which the material has yielded and outside of which still in the elastic state. The elastic-plastic analysis of the thick cylinder can be performed on the basis of the small deformation theory and von Mises yield criterion with the stress-strain relation of the elastic-power law plastic-material model and by separating the cylinder into the elastic region ($c \le r \le b$) and the plastic region ($a \le r \le c$) and solving them.

2.1 Stress, strain, and displacement in the elastic region (c ≤ r ≤ b)

Let pressure acting on the elastic-plastic interface be p_j ,then for the open-ended thick cylinder of inside radius c and outside radius b, subjected to internal pressure p_j, Lame's equation results the stresses:

$$\sigma_\theta = \{p_j c^2/(b^2 - c^2)\}(1 + b^2/r^2) \quad (1)$$

$$\sigma_r = \{p_j c^2/(b^2 - c^2)\}(1 - b^2/r^2) \quad (2)$$

Strains: for incompressible material, Poisson's ratio $\nu = 0.5$ and E = Young's modulus;

$$\varepsilon_\theta = \{p_j c^2/2E(b^2 - c^2)\}(1 + 3b^2/r^2) \quad (3)$$

$$\varepsilon_r = \{p_j c^2/2E(b^2 - c^2)\}(1 - 3b^2/r^2) \quad (4)$$

$$\varepsilon_z = -\{p_j c^2/E(b^2 - c^2)\} \quad (5)$$

Displacement:

$$u = \varepsilon_\theta r = \{p_j c^2/2E(b^2 - c^2)\}r(1 + 3b^2/r^2) \quad (6)$$

Eq. (1) through (6) are the solutions of the elastic region ($c \le r \le b$) of the cylinder. If c and p_j are determined from the boundary conditions, the stresses, strains, and displacement of any position in this region may easily be calculated.

2.2 Stress, strain, and displacement in the plastic region (a ≤ r ≤ c)

According to assumption, the stress-strain relation of a strain-hardened material is defined in [8]:

$$\sigma = E\varepsilon; \sigma \le \sigma_y \quad (7)$$

$$\sigma = A\varepsilon^n; \sigma > \sigma_y \quad (8)$$

Where σ, ε are the equivalent stress and strain; σ_y, ε_y are the material yield strength and strain, respectively, with $\varepsilon_y = \sigma_y / E$ and A is the strength coefficient of material, ($= \sigma_y / \varepsilon_y^{\ n}$) and n is the strain-hardening index of material, $0 \le n \le 1$. Eq. (7) and (8), define an elastic-power law plastic-model to represent the stress-strain relation of a strain-hardened material, with the basic equations abiding the small deformation theory and von Mises yield criterion for determining the solution in the strain-hardened open-ended thick cylinder will be as follows:

The equilibrium equation:

$$r(d\sigma_r/dr) + \sigma_r - \sigma_\theta = 0 \quad (9)$$

The constitutive equations:

$$\sigma = A\varepsilon^n \quad (10)$$

$$\varepsilon_\theta = (\varepsilon/\sigma)(\sigma_\theta - \sigma_r/2) \quad (11)$$

$$\varepsilon_r = (\varepsilon/\sigma)(\sigma_r - \sigma_\theta/2) \quad (12)$$

where σ is determined from equation, (as $\sigma_z = 0$)

$$\sigma = \sqrt{\sigma_\theta^2 - \sigma_\theta\sigma_r + \sigma_r^2} \quad (13)$$

and the compatibility equation:

$$r(d\varepsilon_\theta/dr) + \varepsilon_\theta - \varepsilon_r = 0 \quad (14)$$

with the boundary conditions:

$$\sigma_r|_{r=a} = -p_i \quad (15)$$

$$\sigma_r|_{r=c} = -p_j \quad (16)$$

$$\sigma|_{r=c} = \sigma_y \quad (17)$$

Eq. (9) through (17) consists of the boundary-value problem. As the stresses are unknowns, the problem may be solved as follows: Introducing an auxiliary variable angle ø = ø(r) and considering the new stress functions as:

$$\sigma_\theta = (2/\sqrt{3})\sigma\cos(\phi + \pi/6) \quad (18)$$

$$\sigma_r = -(2/\sqrt{3})\sigma\sin\phi \quad (19)$$

where the Eq. (13) is satisfied by Eq. (18) and (19). Substituting Eq. (18) and (19) into Eq. (11) and (12), it is found that

$$\varepsilon_\theta = \varepsilon\cos\phi \quad (20)$$

$$\varepsilon_r = -\varepsilon\cos(\phi - \pi/3) \quad (21)$$

Substituting Eq. (9), (10), (18), (19), (20) and (21) in Eq. (14), the resulting equation becomes:

$$\frac{d\sigma}{\sigma} = \frac{\sin\phi + \sqrt{3}\cos\phi}{(\cos\phi/n) - \sqrt{3}\sin\phi}d\phi \quad (22)$$

Integrating it from c to r, would result:

$$\sigma = \sigma_y\left[\frac{\sin(\phi_n - \phi_c)}{\sin(\phi_n - \phi)}\right]^{\frac{3n^2+n}{3n^2+1}} e^{\frac{\sqrt{3}n(1-n)}{3n^2+1}(\phi - \phi_c)} \quad (23)$$

where $\phi_c = \sin^{-1}(\sqrt{3}p_j/2\sigma_y)$ (24)

and $\phi_n = \sin^{-1}\left(1/\sqrt{3n^2+1}\right)$ (25)

Eq. (23) is the expression of equivalent stress σ in terms of auxiliary variable angle ø. From this equation the functional relationship between ø and r can be found consequently. Substituting Eq. (18), (19) and (23) into Eq. (9) would result:

$$\frac{dr}{r} = \frac{2n\sin\phi d\phi}{\sqrt{3}n\sin\phi \quad \cos\phi} \quad \frac{2\cos\phi d\phi}{\sin\phi + \sqrt{3}\cos\phi} \tag{26}$$

Integrating this Eq. (26) from a to r would result:

$$r = a\sqrt{\frac{\cos\left(\phi_a \quad \frac{\pi}{6}\right)}{\cos\left(\phi \quad \frac{\pi}{6}\right)}} \left(\frac{\sin(\phi \quad \phi_n)}{\sin(\phi_a \quad \phi_n)}\right)^{\frac{2n}{3n^2+1}} e^{\frac{\sqrt{3}}{2}\frac{n^2 \quad 1}{3n^2+1}(\phi \quad \phi_a)}$$

(27)

As the value of auxiliary variable ø at r = a, is the root of the following equation, resulting from, substituting Eq. (19) and (23) into Eq. (15);

$$\frac{p_i}{\sigma_y} = \frac{2}{\sqrt{3}} \left(\frac{\sin(\phi_n \quad \phi_c)}{\sin(\phi_n \quad \phi_a)}\right)^{\frac{3n^2+n}{3n^2+1}} e^{\frac{\sqrt{3}n(1 \quad n)}{3n^2+1}(\phi_a \quad \phi_c)} \quad \sin\phi_a$$

(28)

Eq. (28) gives the relationship between $ø_a$ and $ø_c$.
To determine $ø_a$ and $ø_c$ separately, another equation consisting of $ø_a$ and $ø_c$ is required, which is found by using the boundary conditions at the elastic-plastic interface r = c are:

$$\sigma_\theta\big|_{r=c} = p_j\left\{\left(b^2 + c^2\right)/\left(b^2 \quad c^2\right)\right\} \tag{29}$$

$$\sigma_r\big|_{r=c} = -p_j \tag{30}$$

Substituting Eq. (29), (30) and (13) in Eq. (17),

$$p_j = \sigma_y\left(b^2 \quad c^2\right)/\sqrt{3b^4 + c^4}$$

(31)

Inserting Eq. (19) into Eq. (16), results:

$$\phi_c = \sin^{\quad 1}\left(\sqrt{3}/2\right)\left(b^2 \quad c^2\right)/\sqrt{3b^4 + c^4}$$

(32)

where c is determined from Eq. (27) as:

$$c = a\sqrt{\frac{\cos\left(\phi_a \quad \frac{\pi}{6}\right)}{\cos\left(\phi_c \quad \frac{\pi}{6}\right)}} \left(\frac{\sin(\phi_c \quad \phi_n)}{\sin(\phi_a \quad \phi_n)}\right)^{\frac{2n}{3n^2+1}} e^{\frac{\sqrt{3}}{2}\frac{n^2 \quad 1}{3n^2+1}(\phi_c \quad \phi_a)}$$

(33)

Substituting Eq. (33) into Eq. (32), the second functional relationship between $ø_a$ and $ø_c$ is obtained. Eq. (28), (32) and (33) can be solved by using iterative procedures.
On determination of $ø_a$, the functional relationship between r and ø will solely be determined from Eq.(28). Using this Eq.(28) with Eq. (18) and (19), the stresses σ_r and σ_θ at any position (r,θ) in the plastic region (a ≤ r ≤ c) will be readily obtained.

Again, by using the expression of the plastic stress components, the expressions of strains in the plastic region (a≤ r ≤ c) can be determined as follows: from Eq. (20) and (21);

$$\varepsilon_\theta\left(\sigma_r \quad \sigma_\theta/2\right) = \varepsilon_r\left(\sigma_\theta \quad \sigma_r/2\right) \tag{34}$$

and substituting Eq. (18) and (19) into this Eq. (34), it results:

$$\varepsilon_r = \quad \left\{\cos\left(\phi \quad \pi/3\right)/\cos\phi\right\}\varepsilon_\theta$$

(35)

and substituting this Eq. (35) into the compatibility Eq. (14), result obtained is:

$$\frac{d\varepsilon_\theta}{\varepsilon_\theta} = \quad \left\{\sqrt{3}\cos\left(\phi \quad \pi/6\right)/\cos\phi\right\}\frac{dr}{r} \tag{36}$$

Substituting Eq. (26) into Eq. (36) and integrating the resulting from $ø_c$ to ø, the expression of strains will finally be found:

$$\varepsilon_\theta = \varepsilon_\theta^j \left(\frac{\sin(\phi_c \quad \phi_n)}{\sin(\phi \quad \phi_n)}\right)^{\frac{3n+1}{3n^2+1}} \frac{\cos\phi}{\cos\phi_c} e^{\frac{\sqrt{3}(1 \quad n)}{3n^2+1}(\phi \quad \phi_c)} \tag{37}$$

$$\varepsilon_r = \quad \left\{\cos\left(\phi \quad \pi/3\right)/\cos\phi\right\}\varepsilon_\theta$$

(38)

$$\varepsilon_z = \quad \left(\varepsilon_r + \varepsilon_\theta\right) \tag{39}$$

where $\varepsilon_\theta^j = \left(\sigma_y/2E\right)\left\{\left(3b^2 + c^2\right)/\sqrt{3b^4 + c^4}\right\}$ (40)

and according to the value of c obtained from Eq. (32).The displacement in the plastic region is obtained as:

$$u = r\varepsilon_\theta = r\varepsilon_\theta^j \left(\frac{\sin(\phi_c \quad \phi_n)}{\sin(\phi \quad \phi_n)}\right)^{\frac{3n+1}{3n^2+1}} \frac{\cos\phi}{\cos\phi_c} e^{\frac{\sqrt{3}(1 \quad n)}{3n^2+1}(\phi \quad \phi_c)}$$

(41)

Eq. (18), (19), (37) through (41) represent the general solution in the plastic region of the open-ended thick cylinder. Once p_j and c are determined from Eq. (31) and (33), respectively, the stress, strain, and displacement at any position(r, θ) is the plastic range (a ≤ r ≤ c) will readily be obtained by using these equations. Similarly, by substituting the values of p_j and c into Eq.(1) through (6), the results for the stress, strain, and displacement at any point (r, θ) in the elastic region ($c \le r \le b$) of the open-ended thick cylinder can also be obtained.

3. VALIDATION

3.1 Compatibility with solution for stresses of an elastic-perfectly plastic-material open-ended thick cylinder

When n=0, Eq. (8) becomes an elastic-perfectly plastic stress-strain relation of material, and Eq. (23) results:

$$\sigma = \sigma_y \tag{42}$$

and Eq.(27) results:

$$r = a\sqrt{\cos(\phi_a \quad \pi/6)/\cos(\phi \quad \pi/6)}\, e^{[(\sqrt{3}/2)(\phi_a \quad \phi)]} \tag{43}$$

where $\phi_a = \sin^{\quad 1}\left(\sqrt{3}p_i/2\sigma_y\right)$ (44)

With Eq. (42) through (44) and Eq. (18), (19), the stresses in the plastic region ($a \le r \le c$) for an elastic-perfectly plastic-open-ended thick cylinder will be:

$$\sigma_\theta = \left(2/\sqrt{3}\right)\sigma_y \cos(\phi + \pi/6)$$

(45)

and $\sigma_r = \quad \left(2/\sqrt{3}\right)\sigma_y \text{ s } \quad \phi$

(46)

and the stresses in the elastic region ($c \le r \le b$) of this subject, with Eq. (1), (2) and (31), will be:

$$\sigma_\theta = \quad \sigma_y c^2 \Big/ \sqrt{3b^4 + c^4} \quad \left(1 + b^2/r^2\right) \tag{47}$$

and $\sigma_r = \quad \sigma_y c^2 \Big/ \sqrt{3b^4 + c^4} \quad \left(1 \quad b^2/r^2\right)$ (48)

where c is obtained from Eq.(43) with Eq.(32):

$$c = a\sqrt{\cos(\phi_a \quad \pi/6)/\cos(\phi_c \quad \pi/6)}\, e^{[(\sqrt{3}/2)(\phi_a \quad \phi_c)]} \tag{49}$$

where $\phi_c = \tan^{\quad 1}\left\{\sqrt{3}\left(b^2 \quad c^2\right)/\left(3b^2 + c^2\right)\right\}$ (50)

The numerical results of expression in Eq. (43), (45) through (49) for stresses are exactly same as those in [6]. So the general solution in this paper is compatible with that in [6].

3.2 Compatibility with solution for strains of an elastic-perfectly plastic-material open-ended thick cylinder

Substituting n = 0, in Eq. (25) and from Eq. (37), (38) and (39), the strains in the plastic region for an open-ended thick cylinder of elastic-perfectly plastic material will be:

$$\varepsilon_\theta = \left(\sigma_y/2E\right)\left\{\left(3b^2 + c^2\right)/\sqrt{3b^4 + c^4}\right\} e^{[\sqrt{3}(\phi \quad \phi_c)]} \tag{51}$$

$$\varepsilon_r = \quad \{\cos(\phi \quad \pi/3)/\cos\phi\}\varepsilon_\theta$$

(52)

$$\varepsilon_z = \quad (\varepsilon_r + \varepsilon_\theta) \tag{53}$$

where c and $ø_c$ are defined in Eq. (49) and (50). The numerical results of these expressions are exactly same with those in [6]. This fact reveals that the general solution in this paper is compatible with that in [6].

3.3 The elastic-limit pressure, p_y

When c = a i.e. the inner surface of the cylinder starts yielding, at an internal pressure which is called the elastic limit pressure, p_y.

For c = a, $ø_c = ø_a$ and from Eq. (31) one get:

$$p_y/\sigma_y = \left(K^2 \quad 1\right)\Big/\sqrt{3K^4 + 1} \tag{54}$$

where K, the radius ratio = b/a and Eq. (54) is the elastic-limit pressure formula of the cylinder from the general solution and is identical with the solution of Lame based on Von Mises yield criterion at r = a [7].

3.4 The plastic-limit (full-yield) pressure, p_u

When c = b i.e. the outer surface of the cylinder has yielded, at an internal pressure which is called the plastic limit or full-yield pressure, p_u.

Substituting c = b into Eq. (28), it would result**:**

$$p_u = \frac{2}{\sqrt{3}}\sigma_y \left[\frac{\sin\phi_n}{\sin(\phi_n \quad \phi_a)}\right]^{\frac{3n^2+n}{3n^2+1}} e^{\frac{\sqrt{3}n(1 \quad n)}{3n^2+1}\phi_a} \sin\phi_a \tag{55}$$

where $ø_a$ is the root of the following Eq.(56):

$$\frac{\sqrt{3}}{2}K^2 = \cos\left(\phi_a \quad \frac{\pi}{6}\right)\left[\frac{\sin\phi_n}{\sin(\phi_a \quad \phi_n)}\right]^{\frac{4n}{3n^2+1}} e^{\sqrt{3}\frac{1 \quad n^2}{3n^2+1}\phi_a} \tag{56}$$

Eq. (55) is the formula for the plastic-limit pressure given by the general solution of this subject. When n = 0 i.e. for an open-ended cylinder of elastic-perfectly plastic-material, Eq.(55) reduces to:

$$p_u\big|_{n=0} = \left(2/\sqrt{3}\right)\sigma_y \sin\phi_a \tag{57}$$

where $ø_a$ is the root of the following Eq.(58):

$$\left(\sqrt{3}/2\right)K^2 = \cos(\phi_a \quad \pi/6)e^{(\sqrt{3}\phi_a)} \tag{58}$$

where K, the radius ratio = b/a. Eq. (57) is the plastic-limit or full-yield-pressure formula for an open-ended thick cylinder of elastic-perfectly plastic-material and the numerical results are exactly same as those in [6].

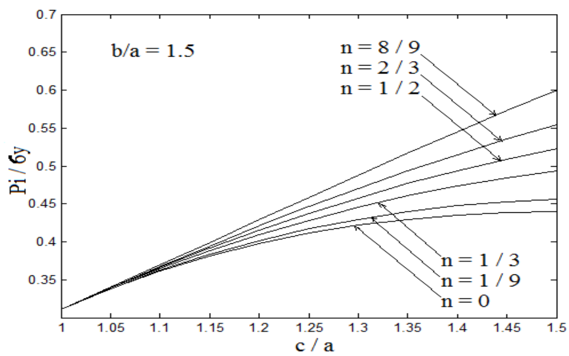

Fig 2. Distribution of p_i / σ_y for different values of c /a and n for b/a =1.5

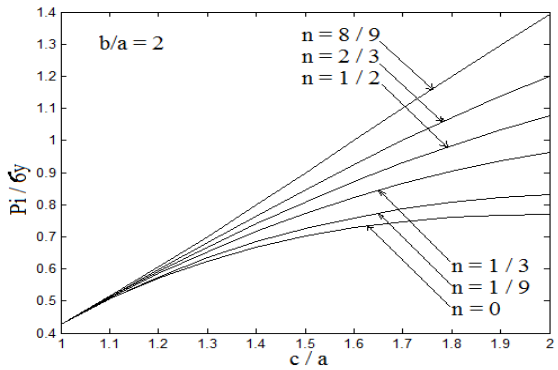

Fig 3. Distribution of p_i / σ_y for different values of c /a and n for b/a =2.0

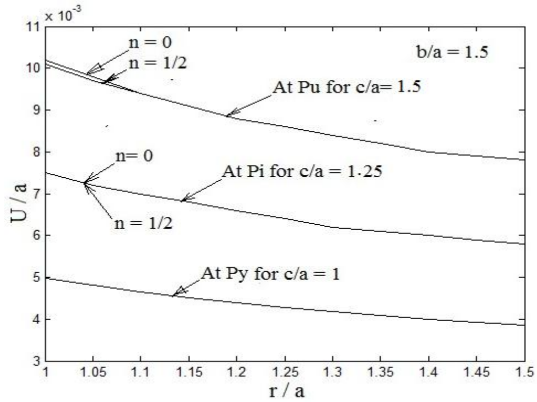

Fig 4. Distribution of u/a for r /a for b/a =1.5 (some specific cases)

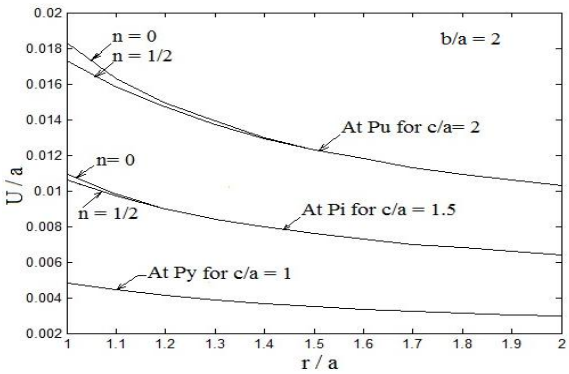

Fig 5. Distribution of u/a for r /a for b/a =2.0 (some specific cases)

4. CONCLUSIONS

Eq. (57), a specific case, reflects the influences of σ_y and K, while, in addition to these, Eq. (55) has effect of n (strain-hardening index), on the limiting load-carrying capacity. The comparisons are shown in fig.2 and 3 for radius ratios, b/a = 1.5 and 2.0, respectively, by solving Eq. (28) and results follow:

(1) The strain-hardening effect (n) has a significant role on the limiting load-carrying capacity. From fig.2 and 3, for radius ratios, b/a = 1.5 and 2.0, respectively, the load-carrying capacity(p_u/σ_y) of an open-ended thick cylinder of the elastic-perfectly plastic-material (n = 0) can be used for an cylinder of a strain-hardened material with a smaller n (say n = 1/9), while for a strain-hardened open-ended thick cylinder with a larger n (say = 8/9, 2/3,..), the strain-hardening effect(n) will be considered to yield a reasonable strength design based on the plastic-limit pressure.

(2) The elastic-limit load-carrying capacity (p_y) is much smaller than that of the plastic-limit (p_u), even for an open-ended thick cylinder of elastic-perfectly plastic-material (n = 0), and as well as for a strain-hardened open-ended thick cylinder with a larger n, for the radius ratios b/a = 1.5 and 2, respectively. Hence, the design based on the elastic-limit analysis underestimates the load-carrying capacity of the strain-hardened open-ended thick cylinder, which results in loss of material.

(3) The load-carrying capacities (p_i/σ_y) of the open-ended thick cylinders of different strain-hardening effects (n) are reasonably different under the same overstrain [say, 50% (c/a = 1.25 for b/a = 1.5 and c/a = 1.5 for b/a = 2)]. The larger the n is, the larger the load- carrying capacity (p_i/σ_y) is.

(4) The distributions of radial displacement (u) are shown in fig.4 and 5, for b/a = 1.5 and 2.0, respectively assuming a strain-hardened material of σ_y = 1070 MPa and E = 207000 MPa and based on p_y = 332.45 MPa, p_i = 440.2MPa for n = 0 and 467.8 MPa for n = 1/2 at c/a = 1.25, and p_u = 470.16 MPa for n = 0 and 559.72 MPa for n = 1/2, for b/a = 1.5; similarly p_y = 458.57 MPa, p_i = 751.36MPa for n = 0 and 865.1 MPa for n = 1/2 at c/a = 1.5, and p_u = 823.66 MPa for n = 0 and 1070.64 MPa for n = 1/2,for b/a = 2.0. Radial displacement u increases as strain-hardening index n decreases and gradually decreases from inside surface to outside surface of the cylinder.

5. REFERENCES

1. Stacey, A., Webster, G. A., 1988, “Determination of residual stress distributions in autofrettaged tubing”, Pressure vessel and piping, 31: 205-20.
2. Lazzaarin, P., Livieri, P., 1997, “Different solutions for stress and strain fields in autofrettaged thick-walled cylinders”, Pressure vessel and piping, 71: 231-38.
3. Zhu, R., Yang, J., 1998, “Autofrettage of thick cylinders”, Pressure vessel and piping, 75: 443-46.
4. [Parker A. P., Huang, X., 2007, “Autofrettage and Reautofrettage of a spherical pressure vessel”, Pressure vessel technology, ASME. 129: 83-88.
5. Chakrabarty, J., 1998, “Theory of Plasticity”,McGraw Hill, New York.

6. Xin-lin, G., 1992, “An exact elasto-plastic solution for an open-ended thick-walled cylinder of a strain-hardening material”, Pressure vessel and piping, 52: 129-144.
7. Harvey, .J. F., 1985, “Theory and design of pressure vessels”, Von Nostrand Reinhold Co.
8. Dieter G. E., 1988, “Mechanical Metallurgy”, McGraw Hill Book Co. Ltd., UK.

6. NOMENCLATURE

Symbol	Meaning	Unit
A	Strength coefficient	---
a	Inside radius	m
b	Outside radius	m
c	Elastic-plastic juncture radius	m
E	Young’s modulus	MPa
e	Exponential term	---
K	Radius ratio (= b/a)	---
n	Strain-hardening index	---
p_i, p_j	Internal pressure, Elastic-plastic juncture pressure	MPa
p_y, p_u	Elastic-limit pressure, Plastic-limit (full yield) pressure	MPa
r	Variable radius	m
u	Radial displacement	m
x, y	Cartesian co-ordinate	m
θ	Tangential co-ordinate	rad
σ, σ_y	Equivalent, Yield stress	MPa
$\varepsilon, \varepsilon_y$	Equivalent, Yield strain	---
$\sigma_\theta, \sigma_r, \sigma_z$	Tangential, radial, axial stress	MPa
$\varepsilon_\theta, \varepsilon_r, \varepsilon_z$	Tangential, radial, axial strain	---
ε_θ^j	Tangential elastic-plastic juncture strain	---
ϕ	Auxiliary angle [ϕ (r)]	rad
ϕ_a , ϕ_c	Auxiliary angle, at r = a, r = c	rad
ϕ_n	Angle of strain-hardening index [ϕ (n)]	rad

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